Online Appendix to *The Collapse of Venezuela: Scorched Earth Politics and Economic Decline, 2012-2020.*

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Appendix to Chapter 3

General Model[i]

Label each party's choice of whether to accept the election's outcome conditional on the results as $A_{ij} \in \{0,1\}$. $A_{ij} = 0$ when party *i* decides to contest *j*'s victory and 1 when it decides to accept it. I will allow for $A_{ii} = 0$, i.e., for party i not to recognize the outcome of its own victory.[ii] The reason is that recognition entails recognizing the distribution of rewards (δ and $1 - \delta$) and a party may decide that it prefers the distribution of rewards that comes out of conflict..

If either party decides not to accept the election result (i.e., $A_{1j}A_{2j} = 0$), then the groups will enter into conflict for the distribution of the endowment, leading group *i* to receive $C(P_i, P_j)R$. Let F_i denote the first derivative of function F with respect to its ith argument, i.e.: $F_i = \frac{\partial F(X_1 \dots X_n)}{\partial X_i}$. I assume q is twice continuously differentiable with $q \in [0,1], q_1 > 0, q_2 < 0, q_{11} < 0,$ and $q(P_i, P_i) = \frac{1}{2}$. Regarding C, I assume that it is also twice continuously differentiable with $C > 0, C_2 < 0, C_1(0, P_j) > 0 \forall P_j \geq 0, C_{11} < 0, C(0,0) = \frac{1}{2}$ and $C(P_i, P_j) + C(P_j, P_i) \leq 1$.[iii]

One key distinction between $C(.)$ and $q(.)$ is that conflict has an adverse impact on available resources. I model this as the requirement that $C(.)$ is strictly decreasing with respect to increases of equal magnitude in P_i and P_j when P_i and P_j are equal.

Assumption 1 *Global destructiveness of conflict:* $C(P_i, P_j) + C(P_j, P_i) < 1$ if $P_i > 0$ and $P_j > 0$.

Assumption 2 *Local destructiveness of conflict:* $C_1(P_i, P_j) + C_2(P_i, P_j) < 0$ for $P_i = P_j \geq 0$.

Note that by construction, electoral competition is neither locally nor globally destructive.[iv] Note also that assumption 2 is weaker than the alternative assumption of homogeneity of degree less than zero. In other words, the results below hold when C is homogeneous of degree less than zero.[v]

Both groups choose E and P to maximize expected utility subject to the constraint

$$E_i + P_i = \overline{L_i}. \tag{A3.1}$$

Given that (A3.1) determines E_i for any P_i, we can write the strategy of each party compactly as the vector $s_i = \{P_i, A_{ii}, A_{ij}\} \in S = [0, \overline{L}] \times \{0,1\}^2$. I solve for subgame perfect Nash equilibria through backward induction given the temporal structure described in figure 3.1.

We can characterize each group's payoff function as:

$$V^i(P,E,A) = \begin{cases} q(E_i,E_j)\delta R + \left(1 - q(E_i,E_j)\right)(1-\delta)R \\ \quad if\ A_{ii}A_{ji} = 1 \cap\ A_{ij}A_{jj} = 1 \\ q(E_i,E_j)C(P_i,P_j)R + \left(1 - q(E_i,E_j)\right)(1-\delta)R \\ \quad if\ A_{ii}A_{ji} = 0 \cap\ A_{ij}A_{jj} = 1 \\ q(E_i,E_j)\delta R + \left(1 - q(E_i,E_j)\right)C(P_i,P_j)R \\ \quad if\ A_{ii}A_{ji} = 1 \cap\ A_{ij}A_{jj} = 0 \\ C(P_i,P_j)R \\ \quad if\ A_{ii}A_{ji} = 0 \cap\ A_{ij}A_{jj} = 0 \end{cases} \tag{A3.2}$$

where $P = \{P_1, P_2\}$, $E = \{E_1, E_2\}$ and $A = \{A_{11}, A_{12}, A_{21}, A_{22}\}$. A subgame perfect Nash equilibrium is a profile of strategies $s = \{s_1, s_2\}$ that is also a Nash equilibrium for the post-election subgame (i.e., the subgame that begins once nature has selected a winner).

I begin with the post-election subgame. Without loss of generality, I will refer to player 1 as the winner and player 2 as the loser of the election when discussing the post-election subgame. I will thus look for the strategy profiles (A_1, A_2) that constitute Nash equilibria in this subgame (where I write A_{11} as A_1 and A_{21} as A_2 given our assumption that 1 has won). Let a **contested election** refer to an outcome for which $A_1A_2 = 0$ and an **uncontested** election as one for which $A_1A_2 = 1$. Let $V^1(A_1, A_2)$ and $V^2(A_2, A_1)$ denote respectively the payoffs of the winner and the loser depending on each player's decision on whether to recognize the result or not:

$$V^1(A_1,A_2) = \begin{cases} \delta R & \text{if } A_1A_2 = 1 \\ C(P_1,P_2)R & \text{if } A_1A_2 = 0 \end{cases} \tag{A3.3}$$

$$V^2(A_2,A_1) = \begin{cases} (1-\delta)R & \text{if } A_1A_2 = 1 \\ C(P_2,P_1)R & \text{if } A_1A_2 = 0 \end{cases} \tag{A3.4}$$

Note that if the loser accepts the results ($A_2 = 1$) it will be a best response for the winner to accept the results ($A_1 = 1$) when:

$$C(P_1, P_2) \leq \delta \tag{A3.5}$$

while the winner accepts the results ($A_1 = 1$), it will be a best response for the loser to accept the results ($A_2 = 1$) when:

$$C(P_2, P_1) \leq (1 - \delta). \tag{A3.6}$$

However, if the loser contests the results, then the winner's payoff will be $C(P_1, P_2)$ regardless of whether they accept or contest, making both $A_1 = 1$ and $A_1 = 0$ best responses to $A_2 = 0$. This introduces the possibility that the strategy profile $(A_1, A_2) = (0,0)$ could be a Nash equilibrium regardless of whether (A3.5) or (A3.6) hold. However, note that if (A3.5) holds as a strict inequality, then $A_1 = 0$ will be a weakly dominated strategy and if (A3.6) holds as a strict inequality, $A_2 = 0$ will also be weakly dominated. In order to eliminate this possibility, we will require that pure-strategy Nash equilibria also be *trembling hand perfect* in the sense of Selten (1975), that is, that they also be Nash equilibrium in a perturbed game in which all pure strategies are played with an infinitesimally small probability. Note that if either (A3.5) or (A3.6) hold as strict inequalities, then $(0,0)$ is not trembling hand perfect.

Proposition 0. *Both sides accepting the result of the election with probability 1 will be a trembling-hand perfect Nash equilibrium of the post-election subgame if and only if equations (A3.5) and (A3.6) hold. Although both sides contesting with probability 1 will also be a Nash* *equilibrium, it will not be trembling-hand perfect if (A3.5) and (A3.6) hold. If either (A3.5) or (A3.6) fail to hold, then the only Nash equilibria of the post-election subgame will be ones in which at least one of the sides contests the election with probability 1.*

Proof: *Since accepting is a best response for both the winner and loser if their opponent accepts when (A3.5) and (A3.6) hold, then (1,1) is a Nash equilibrium. Let* $C^i = C(P_i, P_j)$. *The payoff for 1 from accepting if 2 accepts in an* ϵ*-perturbed game is* $\delta(1-\epsilon)+\epsilon C^1$ *which is greater than or equal to* $C^1(1-\epsilon)+\epsilon C^1$*, the payoff from contesting, if (A3.5) holds. Similarly,* $(1-\delta)(1-\epsilon)+\epsilon C^2 \geq C^2(1-\epsilon)+\epsilon C^2$ *is (A3.6) holds, so (1,1) is trembling-hand perfect if (A3.5) and (A3.6) hold. (0,0) will not be trembling-hand perfect if either (A3.5) or (A3.6) hold because it involves at least one player playing a weakly dominated strategy. If (A3.5) fails to hold, then the best response for the winner will be to contest if the loser accepts, and contesting will always be a best response for either side if the other side contests. Therefore, the only case in which it accepting would not be a strictly dominated strategy for the winner will be if 2 contests. Thus, if 1 is playing a best response, at least one of the players must be contesting. A similar reasoning holds when (A3.6) fails to hold.*■

Corollary 0 *If both (A3.5) and (A3.6) hold but only one of them holds as a strict equality, then there will be another pure-strategy trembling-hand perfect Nash equilibrium in which the side that is indifferent between conflict and the election contests the result while the other side accepts it. In addition, there will be a continuum of mixed-strategy Nash equilibria in which the side that is indifferent between conflict and the election randomizes with any probability* $p \in (0,1)$.

Proof: *Assume (A3.5) holds as strict inequality and (A3.6) as an equality. For player 1 accepting is a best response to 2 accepting but both actions are best responses to 2 contesting.*

Thus (1,0) is also a Nash equilibrium. Player 1's payoff from accepting in an ϵ-perturbed game is $(1-\epsilon)C^1+\epsilon\delta$ *which is greater than or equal to* $(1-\epsilon)C^1+\epsilon C^1$*, the payoff from contesting. For player 2, the payoff from contesting in an ϵ-perturbed game is* $(1-\epsilon)C^2+\epsilon C^2$ *which is equal to* $(1-\epsilon)(1-\delta)+\epsilon C^2$*, the payoff from accepting. Thus (1,0) is trembling hand perfect. In the mixed strategy equilibrium (1,p), player 2's payoff in an ϵ-perturbed game* $(1-\epsilon)[p(1-\delta)+(1-p)C^2]+\epsilon C^2=1-\delta=C^2$*, so that he continues to be indifferent between randomizing and playing any strategy. Since 1 is facing a mixed strategy, its selection is also optimal at* $\mathrm{p}=\epsilon$*, confirming that (1,p) is trembling-hand perfect.*

Proposition 0 allows us to rewrite (12) as:

$$V^i(P,E,A)=\begin{cases} q(E_i,E_j)\delta R+\left(1-q(E_i,E_j)\right)(1-\delta)R \\ \text{if } C(P_i,P_j)\le 1-\delta \text{ and } C(P_j,P_i)\le 1-\delta \\ q(E_i,E_j)C(P_i,P_j)R+\left(1-q(E_i,E_j)\right)(1-\delta)R \\ \text{if } C(P_i,P_j)\le 1-\delta \text{ and } C(P_j,P_i)>1-\delta \\ q(E_i,E_j)\delta R+\left(1-q(E_i,E_j)\right)C(P_i,P_j)R \\ \text{if } C(P_i,P_j)>1-\delta \text{ and } C(P_j,P_i)\le 1-\delta \\ C(P_i,P_j)R \\ \text{if } C(P_i,P_j)>1-\delta \text{ and } C(P_j,P_i)>1-\delta \end{cases} \tag{A3.7}$$

Let P^* denote the choice of P by both parties in a pure-strategy SSPNE where the election is contested and P^{**} the choice of P when it is contested. We are now ready to establish

Proposition 1 *In any pure-strategy symmetric subgame perfect Nash equilibrium,*

$$C(P^*,P^*)=1-\delta \tag{A3.8}$$

if the election is uncontested and

$$P^{**} = \underset{P_i \in [0,\overline{L_i}]}{Argmax} [C(P_i, P^{**})] \qquad (A3.9)$$

if the election is contested.

Proof. *That equation (A3.6) holds in an uncontested SSPNE follows directly from Proposition 0. First consider the case in which* $P^* = 0$*. Since* $C(0,0) = \frac{1}{2}$ *and* $1 - \delta \leq \frac{1}{2}$*, then the only case in which* $P^* = 0$ *can correspond to an uncontested SSPNE is if* $\delta = \frac{1}{2}$*. In that case,* $C(0,0) = 1 - \delta$ *and (A3.6) holds as equality. Now consider cases in which* $P^* > 0$*. If the election is uncontested,* $A_{12} = A_{21} = A_{11} = A_{22} = 1$ *and* $V_i = q(E_i, E_j)\delta R + \left(1 - q(E_i, E_j)\right)(1 - \delta)R$*. But then if (A3.6) were to hold as a strict inequality and* $P^* > 0$*, group i could raise* E_i *and lower* P_i *by an infinitesimally small amount and increase its payoff, as* $V_i = R(2\delta - 1)q_1 > 0$*. It follows that (A3.6) must hold as an equality and* $C(P^*, P^*) = 1 - \delta$*. If the election is contested then (A3.6) does not hold and* $C(P^*, P^*) > 1 - \delta$ *so that* $A_{12} = A_{21} = 0$*. Given that each actor's payoff is given by* $C(P_i, P_j)$*,* P_i *must be maximizing this function given* P_j*; if it were not, then it would be possible to improve payoffs by deviating from* P^* *infinitesimally in the direction of the function's positive gradient and still comply with the strict inequality* $C(P_i, P_j) > 1 - \delta$*.*

Corollary 1 *Along any pure-strategy SSPNE where the election is uncontested,* P^* *will be an increasing function of δ. Furthermore, there will be a level δ such that if* $\delta > \overline{\delta}$ *there will be no SSPNE where the election is uncontested.*

Proof. *Differentiating* $C(P^*, P^*) = 1 - \delta$ *gives us* $(C_1 + C_2)dP^* = -d\delta \rightarrow \frac{dP^*}{d\delta} = -\frac{1}{C_1 + C_2} > 0$ *by Assumption 2. Let* $\delta = 1 - C(\overline{L}, \overline{L})$*. Then it follows that if* $\delta > \overline{\delta}$*,* $C(P, P) > 1 - \delta$ *for all* $P \leq \overline{L}$*, making an uncontested SSPNE not feasible.*

Corollary 2 *Along any pure-strategy SSPNE where the election is contested, P^{**} will be independent of δ. Furthermore, there will be a level $\underline{\delta}$ such that if $\delta < \underline{\delta}$, there will be no SSPNE where the election is contested.*

Proof. *The first part follows from the fact that $P^{**} = \underset{P_i \in [0,\overline{L_i}]}{Argmax} [C(P_i, P^{**})]$ is independent of δ. Let $\underline{\delta} = 1 - C(P^{**}, P^{**})$. If $\delta < \underline{\delta}$, then $C(P^{**}, P^{**}) < 1 - \delta$ and the loser has no incentive to contest the election.*

Corollary 3 *There exists a $\tilde{\delta} \in (\frac{1}{2}, 1)$ such that if $\delta < \tilde{\delta}$, there is no pure strategy SSPNE.*

Proof. *Assume that $P^*$is an uncontested election equilibrium for $\delta = \frac{1}{2} + \epsilon$. Then by Proposition 2, $C(P^*, P^*) = \frac{1}{2} - \epsilon$. If $P^*$is an SSPNE, then there is no $P' | V\left(P', P^*, A(P', P^*)\right) > V\left(P^*, P^*, A(P^*, P^*)\right)$.Consider a small increase by player i in P from P^*. If $C_1 > 0$, then i will now contest the result if she loses, as C will now be strictly higher than $1 - \delta$. Because $C_2 < 0$, then j will continue to accept the result if i wins. Therefore, we are on the third segment of (A3.7). Since V is the same at (P^*, P^*) on the first and third segments of (A3.7), then we can assess the increase in utility by calculating $\frac{dV^i}{dP_i}$ along the third segment. In order for to be an SSPNE, this increase must be less than or equal to zero, i.e.:*

$$\frac{dV^i}{dP_i} = -q_1(\delta - C) + (1 - q)C_1 = -q_1(2\delta - 1) + (1 - q)C_1 \leq 0$$

(A3.10)

As $\delta \rightarrow \frac{1}{2}$*,* $\frac{dV^i}{dP_i} \rightarrow \frac{1}{2}C_1(0,0) > 0$*. Thus, (A3.10) cannot hold and* P^* *cannot be an SSPNE. By Corollary 3 we know that* P^{**} *cannot be an SSPNE either, so there is no SSPNE for* $\delta = \frac{1}{2}$*. Alternatively, let* $\delta \rightarrow 1$*. Then* $C > Lim(1-\delta) = 0$ *ensures that the loser will never recognize the result and that there will be conflict independently of who wins. Thus* $V^i = C$ *and there is a SSPNE at* P^{**}*.*■

Proposition 2 *A (possibly mixed-strategy) SSPNE exists.*

Assume $P^* > \overline{L}$*. Then* $\delta > \overline{\delta}$ *and, by Corollary 2, there is no uncontested SSPNE. However, since* P^{**} *is a continuous function from a nonempty convex compact subset of a Euclidean space to itself, then by Brouwer's fixed-point theorem, there exists a* $P^{**} = \underset{P_i \in [0,\overline{L_i}]}{Argmax}[C(P_i, P^{**})] \leq \overline{L}$ *and, by global destructiveness of conflict,* $C(P^{**}, P^{**}) > C(P^*, P^*) = 1-\delta$*, making* P^{**}*a contested SSPNE. Assume instead* $P^* \leq \overline{L}$*. First, consider the case when* $C_1(P^*, P^*) < 0$*.* $C_{11} < 0$ *implies that for any* $P' > P^*$*,* $C_1(P^*, P^*) < 0 \rightarrow C(P', P^*) < C(P^*, P^*) = 1-\delta$*. Thus at* (P', P^*) *player i continues to recognize j's victory, while by* $C_2 < 0$*, player j also continues to recognize j's victory. But then the payoffs will continue to be determined by the first segment of (30), along which* $\frac{dV^i}{dP_i} = q_1(1-2\delta) < 0$*, so that there is no deviation to a* $P' > P^*$ *that can raise i's payoff. Thus consider a deviation to a lower level* $P' < P^*$*. Note that by* $C_2 < 0$*, player j will contest i's victory at* (P', P^*)*, so that i's payoff will be either* $C(P', P^*)$ *or* $q'C(P', P^*) + (1-q')(1-\delta)$*. Yet since* $P' < P^*$*,* $C(P', P^*) < \frac{1}{2}$*, i's payoff at* P' *cannot be higher than at* P^**. As there is no deviation from* P^* *that can improve i's payoff, then* P^* *is an SSPNE. Consider alternatively the case* $C_1(P^*, P^*) > 0$*. We will establish by construction that whenever that is the case, there is an SSPNE in mixed strategies. Note that if* $C_1(P^*, P^*) > 0$*, there must* *exist an interval* (P^*, P'') *such that if* $P \epsilon (P^*, P'')$ *then* $C(P, P^*) > C(P^*, P^*) = 1 - \delta$. *Choose a* $P^+ \epsilon (P^*, P'') | C(P^+, P^*) < \delta$. *Assume that players play* P^**with probability* h and P^+ with probability *1-h*. *Assume also that both players play* $A_1 = A_2 = 1$ *in all cases except when the loser played* P^+*and the winner played* P^*, *in which case the loser contests the election (*$A_2 = 0$*) while the winner accepts it (*$A_1 = 1$*). It is straightforward to verify that these strategies will be trembling-hand perfect equilibria of the post-election subgame. Define* $H^{xy} = H(P^x, P^y)$. *If* $P^1 = P^2 = P^+$, *the fact that* $C^{++} < 1 - \delta$ *ensures that both the winner and loser will accept. If* $P^1 = P^2 = P^*$, *then the loser is indifferent between accepting and contesting while the winner strictly prefers accepting as* $C^{**} = 1 - \delta < \delta$. *If* $P^1 = P^+, P^2 = P^*$ *then* $C^{*+} < 1 - \delta$ *ensures that the loser will not contest and* $C^{+*} < \delta$ *that neither will the winner. If* $P^1 = P^*, P^2 = P^+$ *then* $C^{+*} > 1 - \delta$ *ensures that the loser will contest while* $C^{*+} < \delta$ *ensures the winner will accept. Using* $q^{**} = q^{++} = \frac{1}{2}$ *and* $q^{+*} = 1 - q^{*+}$, *we can write politician payoffs as:*

$$V_i = (h_i h_j)\left(\frac{1}{2}\right) + \left((1 - h_i)(1 - h_j)\right)\left(\frac{1}{2}\right) + h_i(1 - h_j)\left(q^{*+}C^{*+} + (1 - q^{*+})(1 - \delta)\right) +$$
$$(1 - h_i)h_j\left((1 - q^{*+})\delta + q^{*+}C^{*+}\right) \qquad (A3.11)$$

Setting the first derivative of this term equal to zero gives:

$$h_j = h^* = \frac{\delta - \frac{1}{2} + q^{*+}(1 - \delta - C^{*+})}{q^{*+}(1 + C^{+*} - C^{*+})}. \qquad (A3.12)$$

$C^{*+} < 1 - \delta$ *and that* $C^{+*} > C^{*+}$ *ensure that both the numerator and the denominator of (A2.3) are positive, while* $C^{+*} > 1 - \delta$ *and* $q^{*+} > \frac{1}{2}$ *ensure that the numerator is smaller than the denominator, establishing* $h_j \epsilon (0,1)$. *Therefore, at* $h_i = h_j = h^*$, *both parties are indifferent to changing the level of h, making this a subgame perfect equilibrium.*■

Remark 1 *The following condition is necessary for a pure-strategy uncontested SSPNE to exist:*

$$\left.\frac{\partial V^i(P_i, P^*)}{\partial P_i}\right|_{P_i=P^*} = -q_1(\delta - C) + (1-q)C_1 = -q_1(2\delta - 1) + (1-q)C_1 \leq 0$$

(A3.17)

Proof. *Let P^* be an uncontested SSPNE. Then either $C_1 < 0$, in which case (A3.17) holds, or $C_1 > 0$. If $C_1 > 0$ and (A3.17) does not hold, increasing P_i slightly will raise i's payoff, as i will stop recognizing j's victory but j will continue recognizing i's victory, putting the player on the third segment of (A3.7) that (A3.17) describes the first derivative of. So (A3.17) is necessary for there not to be an optimal deviation from P^*.* ■

Note that while (A3.17) is necessary, it may not be sufficient. In general, it is hard to come up with an intuitive sufficient condition for P^* to be an uncontested SSPNE with $C_1 > 0$. One possibility is combining (A3.17) with

$$\frac{\partial^2 V^i(P_i, P^*)}{\partial {P_i}^2} = q_{11}(\delta - C) + 2q_1 C_1 + (1-q)C_{11} < 0 \qquad \text{(A3.18)}$$

Where it is important to note that (A3.18) needs to hold at any $P_i > P^*$, whereas (A3.17) need only hold at $P_i = P^*$. However, (A3.18) may be unnecessarily stringent and in practice (A3.17) will often ensure existence of an uncontested SSPNE for $C_1 > 0$ even if (A3.18) does not hold.

Remark 2: $\tilde{\delta} = Min(\underline{\delta}, \hat{\delta})$, *where $\hat{\delta}$ is given by the lowest solution to the following system of equations:*

$$\hat{\delta} = 1 - C(\tilde{P}, \tilde{P}) \tag{A3.19}$$

$$1 = 2C(\tilde{P}, \tilde{P}) + \frac{C_1(\tilde{P}, \tilde{P})}{2q_1(\overline{L} - \tilde{P}, \overline{L} - \tilde{P})} \tag{A3.20}$$

Proof: *Consider any $\delta' < \hat{\delta}$ and let $P^*(\delta')$ be the corresponding level of P^* defined by (A3.8). We know that at $\delta' = \frac{1}{2}$, (A3.17) does not hold (see proof of Corollary 3). But then it cannot hold at any $\delta' < \hat{\delta}$ given that (A3.19) and (A3.20) are continuous in δ and that if (A3.17) holds at δ' then it must hold as a strict equality for some $\delta'' < \delta'$, making δ'' and $P^*(\delta'')$ a solution to (A3.19) and (A3.20) and contradicting the assumption that $\hat{\delta}$ is the lowest solution to that system of equations.*■

Specific functional form assumptions

I now specialize to particular functional form assumptions that allow us to derive more specific results. Namely, I assume that the conflict and electoral contest functions are:

$$C(P_i, P_j) = \begin{cases} \dfrac{P_i^\gamma}{P_i^\gamma + P_j^\gamma} e^{-\alpha(P_i + P_j)} & \text{if } P_i + P_j > 0 \\ \dfrac{1}{2} & \text{otherwise} \end{cases} \tag{A3.21}$$

$$q(P_i, P_j) = \begin{cases} \dfrac{E_i^\eta}{E_i^\eta + E_j^\eta} & \text{if } E_i + E_j > 0 \\ \dfrac{1}{2} & \text{otherwise} \end{cases} \tag{A3.22}$$

with $\alpha > 0, \gamma \in (0,1], \eta \in (0,1]$. The first part of the functional form for C, $\frac{P_i^\gamma}{P_i^\gamma + P_j^\gamma}$, is the power form of the contest-success function originally proposed by Tullock (1980) for models of rent-seeking and whose use is now standard in the economics of conflict (Skaperdas and Garfinkel 2007). The functional form for C combines this standard formulation with the exponential term $e^{-\alpha(P_i+P_j)}$ which captures the destructiveness of conflict. The functional form for q also uses the power form but has no additional exponential term, capturing the idea that electoral competition, unlike conflict, is not destructive.

Using these functions, we can derive P^* and P^{**} respectively as:

$$P^* = -\frac{\ln\big(2(1-\delta)\big)}{2\alpha} \tag{A3.23}$$

$$P^{**} = \begin{cases} \frac{\gamma}{2\alpha} & \text{if } \frac{\gamma}{2\alpha} < \bar{L} \\ \bar{L} & \text{if } \frac{\gamma}{2\alpha} \geq \bar{L} \end{cases} \tag{A3.24}$$

and

$$\underline{\delta} = \begin{cases} 1 - \frac{1}{2}e^{-\gamma} & \text{if } \frac{\gamma}{2\alpha} < \bar{L} \\ 1 - \frac{1}{2}e^{-\alpha 2\bar{L}} & \text{if } \frac{\gamma}{2\alpha} \geq \bar{L} \end{cases}, \tag{A3.25}$$

$$\overline{\delta} = 1 - \frac{1}{2}e^{-\alpha 2\bar{L}} \tag{A3.26}$$

while:

$$\tilde{\delta} = Min(\underline{\delta}, \hat{\delta}), \tag{A3.27}$$

and

$$\hat{\delta} = 1 - \frac{1}{2}e^{-\alpha 2\tilde{P}}, \tag{A3.28}$$

where $\tilde{P}$ is the solution to:

$$1 + \frac{\frac{\gamma}{2\tilde{P}} - \alpha}{\frac{\eta}{(\bar{L} - \tilde{P})}} = e^{\alpha 2\tilde{P}}. \tag{A3.29}$$

Note that $\underline{\delta} = \overline{\delta}$ if $\bar{L} \leq \frac{\gamma}{2\alpha}$ and $\underline{\delta} < \overline{\delta}$ if $\bar{L} > \frac{\gamma}{2\alpha}$. Note also that by (A3.27) $\tilde{\delta} \leq \underline{\delta}$. Therefore, in this case there are four non-overlapping regions of δ, each associated with different possible pure-strategy equilibria: (i) $\left[\frac{1}{2}, \tilde{\delta}\right)$ for which there is no pure-strategy SSPNE but there is a mixed strategy SSPNE given by (A3.15); (ii) $\left[\tilde{\delta}, \underline{\delta}\right)$, for which there is a single pure-strategy uncontested SSPNE where P^* is given by (A3.8); (iii) $\left[\underline{\delta}, \overline{\delta}\right]$, for which there are two pure-strategy SSPNE: an uncontested one where P^* is given by (A3.8) and a contested one where P^{**} is given by (A3.9); and (iv) $\left(\overline{\delta}, 1\right]$ where there is a single pure-strategy contested SSPNE where P^{**} is given by (A3.9). Note also that when $\bar{L} \leq \frac{\gamma}{2\alpha}$, $\underline{\delta} = \overline{\delta}$ and interval (iii) collapses to a single point.

Note that by equation (A3.27), the relationship between $\tilde{\delta}$ and $\bar{L}$ must be given by:

$$\frac{d\tilde{\delta}}{d\bar{L}} = \begin{cases} \frac{d\underline{\delta}}{d\bar{L}} \text{ if } \hat{\delta} > \underline{\delta} \\ \frac{d\hat{\delta}}{d\bar{L}} \text{ if } \hat{\delta} < \underline{\delta} \end{cases} \tag{A3.30}$$

which, using (A3.25), (A3.28) and (A3.29), becomes:

$$\frac{d\tilde{\delta}}{d\bar{L}} = \begin{cases} 0 \text{ if } \hat{\delta} > \underline{\delta} \text{ and } \frac{\gamma}{2\alpha} < \bar{L} \\ \alpha e^{-\alpha 2\bar{L}} \text{ if } \hat{\delta} > \underline{\delta} \text{ and } \frac{\gamma}{2\alpha} \geq \bar{L} \\ \alpha e^{-\alpha 2\tilde{P}} \frac{-\frac{1}{\eta}\left(\frac{\gamma}{2\tilde{P}}+\alpha\right)}{-\frac{1}{\eta}\left(\frac{\bar{L}\gamma}{2\tilde{P}^2}-\alpha\right)-2\alpha e^{-\alpha 2\tilde{P}}} \text{ if } \hat{\delta} < \underline{\delta} \end{cases} \quad \text{(A3.31)}$$

The second term in equation (A3.31) is positive. Regarding the third term, note that it has a negative numerator. Its denominator will also be negative if $\tilde{P}^2 < \frac{\bar{L}\gamma}{2\alpha}$. Yet if $\hat{\delta} < \underline{\delta}$ then either $\bar{L} \leq \frac{\gamma}{2\alpha}$ in which case $\hat{\delta} = 1 - \frac{1}{2}e^{-\alpha 2\tilde{P}} < \underline{\delta} = 1 - \frac{1}{2}e^{-\alpha 2\bar{L}}$ implies $\tilde{P} < \bar{L} \leq \frac{\gamma}{2\alpha}$ or $\bar{L} > \frac{\gamma}{2\alpha}$, in which case case $\hat{\delta} = 1 - \frac{1}{2}e^{-\alpha 2\tilde{P}} < \underline{\delta} = 1 - \frac{1}{2}e^{-\gamma}$ implies $\tilde{P} < \frac{\gamma}{2\alpha} < \bar{L}$. Therefore $\tilde{P}^2 < \frac{\bar{L}\gamma}{2\alpha}$ is ensured in both cases, ensuring that the third expression in (A3.31) is also positive. This establishes that $\frac{d\tilde{\delta}}{d\bar{L}} \geq 0$.

Appendix to Chapter 7

I assume an economy in which all labor is employed in the non-tradables sector (see chapter 2 for further discussion of this assumption) and labor markets are competitive. Non-tradables production is carried out using a Cobb-Douglas production function $y = AL^{1-\alpha}$. The economy has a fixed endowment of labor. The wage rate will equal the value of the marginal product of labor in non-tradables:

$$W = P_N A(1-\alpha)L^{-\alpha} \quad \text{(A7.1)}$$

Letting γ denote the share of non-tradable goods in workers' consumption basket, real wages will be:

$$w = \frac{W}{P} = \frac{W}{P_N^{\gamma} P_T^{1-\gamma}} = \left(\frac{P_N}{P_T}\right)^{1-\gamma} A(1-\alpha)L^{-\alpha}, \quad \text{(A7.2)}$$

so that real wages are an increasing function of the relative price of non-tradables.

The real exchange rate in turn will be:

$$q = E\frac{P_N^{\gamma}P_T^{1-\gamma}}{P_N^{*\beta}P_T^{*1-\beta}}. \tag{A7.3}$$

Assuming the law of one price holds for tradable goods, $P_T^* = EP_T$. After normalizing $\frac{P_N^*}{P_T^*} = 1$, then equation (A7.3) reduces to

$$q = \left(\frac{P_N}{P_T}\right)^{\gamma} \tag{A7.4}$$

Which can be substituted in (A7.2) to get:

$$w = q^{\frac{1-\gamma}{\gamma}}A(1-\alpha)L^{-\alpha}. \tag{A7.5}$$

Given that the endowment of labor is fixed, this means that real wages have a one-to-one relationship with the real exchange rate. I use the Central Bank of Venezuela's share of non-tradable product in the Consumer Price Index (CPI) basket, 52.1%, as my estimate of γ, which implies that movements in the equilibrium real exchange rate will translate into movements in real wages with elasticity $\frac{1-\gamma}{\gamma} = 0.92$.

Alternatively, assume that the government sets the minimum wage to target a real wage of $\overline{w}$. Then employment will adjust for (A7.5) to hold. This allows us to estimate the employment level in a fixed real wage scenario as:

$$L = \left(\frac{\overline{w}}{A(1-\alpha)}\right)^{-\frac{1}{\alpha}}q^{\frac{1-\gamma}{\alpha\gamma}}. \tag{A7.6}$$

Using an estimate of α=0.7[vi] as well as my previous estimate of γ, this implies that movements in the equilibrium real exchange rate will translate into employment levels with an elasticity $\frac{1-\gamma}{\gamma} = 1.31$.

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[i] The proofs in this section are reproduced from Rodríguez and Imam (2022).

[ii] Unless otherwise stated, I will use the label A_{ij} to refer to the decision of agent i to recognize the victory of her opponent and retain A_{ii} for the decision of agent i to recognize her own victory.

[iii] Note that I do not assume $C_1 > 0$ globally because the destructive effect of conflict can lead there to be a range over which greater spending on conflict, while allowing i to capture a greater share of the pie, may destruct so much output so as to generate a marginal decline in i's payoff.

[iv] This is because the probability that group i wins the election is equal to the probability that group j loses it, i.e. $q(E_i, E_j) = 1 - q(E_j, E_i)$. Differentiating this expression with respect to E_i tells us that $q_1(E_i, E_j) = -q_2(E_j, E_i) = -q_2(E_i, E_j)$ when $E_i = E_j$, so that $q_1(E_i, E_j) + q_2(E_i, E_j) = 0$.

[v] If C is homogeneous of degree r, then by Euler's theorem, $C_1P_1 + C_2P_2 = rC$. If $P_1 = P_2 = P > 0$, then C1 + $C_1 + C_2 = \frac{rC}{P} \leq 0$ as $r \leq 0$.

[vi] I use α=0.7, a conventional assumption in macroeconomic models, instead of the lower α=0.42 labor share for this period reported in Venezuela's national accounts. Venezuela's high labor share is a consequence of the high capital intensity as well as the size of its oil sector. In the models presented in chapter 4, this contribution is separated out as that of an additional resource sector. Regrettably, recent detailed factor share data by sector of the economy are unavailable for Venezuela, impeding us from more precisely estimating the non-oil labor share. Gollin (2002) provides evidence that after adjusting for self-employment, labor shares estimates do not vary systematically with income and average 0.65-0.75, depending on the specifics of the adjustment method.